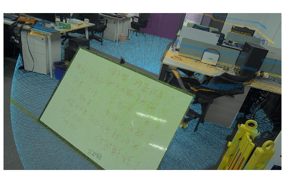
Supplementary

I. Supplement Experiment

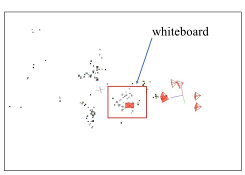
If our sensor suite is mounted on the vehicle toward the ground, Yuan's approach [4] will fail, and our method cannot find a unique solution only using the data that observes the parallel ground or the vertical planes. However, we can find some inclined planes to calibrate. There are many scenes we can find in our daily life, as shown in Fig. 1.

Fig. 1: The useful calibration scenes when the sensor suite is mounted on the vehicle

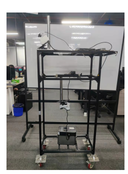
We can also use the whiteboard to calibrate. Just place it near the ground, as shown in Fig. 2. We simulate the situation of mounting the sensor suite on the vehicle. We only move the mobile platform parallel to the ground without changing the relative transformation from sensor suite to the platform. Seven pairs of images and point clouds are collected.



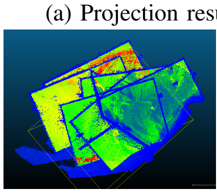
(a) Projection result



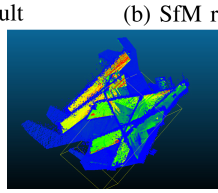
(b) SfM result



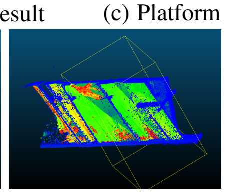
(c) Platform



(d) Front view



(e) Top view



(f) Left view

Fig. 2: Calibration data and result using whiteboard. To clearly visualize the multiple planes' points , we manually cropped this plane out. In the extrinsic calibration, we used the raw data

II. The Jacobian of The Loss Function

According to the chain rule, the jacobian matrix of the residual function $\mathbf{e}_k(\mathbf{T}_{c_i}, \mathbf{T}_{c_i}, \mathbf{T}, s)$ is calculated by:

$$[\mathbf{J}^{\mathbf{R}}_{c_i,c_j,k}, \mathbf{J}^{\mathbf{t}}_{c_i,c_j,k}] = [\frac{\partial \mathbf{p}_k^{c_i}}{\partial \mathbf{R}_l^c}\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{p}_k^{c_i}}\frac{\partial \mathbf{e}_k}{\partial \mathbf{p}_k^{c_j}}, \frac{\partial \mathbf{p}_k^{c_i}}{\partial \mathbf{t}_l^c}\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{p}_k^{c_i}}\frac{\partial \mathbf{e}_k}{\partial \mathbf{p}_k^{c_j}}] \quad (1)$$

$$[\mathbf{J}^{\mathbf{R}_{c_i}}_{c_i,c_j,k}, \mathbf{J}^{\mathbf{t}_{c_i}}_{c_i,c_j,k}] = [\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{R}_{c_i}}\frac{\partial \mathbf{e}_k}{\partial \mathbf{p}_k^{c_j}}, \frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{t}_{c_i}}\frac{\partial \mathbf{e}_k}{\partial \mathbf{p}_k^{c_j}}] \quad (2)$$

$$[\mathbf{J}^{\mathbf{R}_{c_j}}_{c_i,c_j,k}, \mathbf{J}^{\mathbf{t}_{c_j}}_{c_i,c_j,k}] = [\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{R}_{c_j}}\frac{\partial \mathbf{e}_k}{\partial \mathbf{p}_k^{c_j}}, \frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{t}_{c_j}}\frac{\partial \mathbf{e}_k}{\partial \mathbf{p}_k^{c_j}}] \quad (3)$$

$$\mathbf{J}^{s}_{c_i,c_j,k} = \frac{\partial \mathbf{p}_k^{c_j}}{\partial s}\frac{\partial e_{c_i,c_j,k}}{\partial \mathbf{p}_k^{c_j}} \quad (4)$$

$$[\mathbf{J}^{\mathbf{u}_k^{c_i}}_{c_i,c_j,k}, \mathbf{J}^{\mathbf{u}_k^{c_j}}_{c_i,c_j,k}] = [\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{u}_k^{c_i}}\frac{\partial \mathbf{e}_k}{\partial \mathbf{p}_k^{c_j}}, \mathbf{I}_{2\times 2}] \quad (5)$$

$$\mathbf{J}^{\mathbf{n}_{l_i}}_{c_i,c_j,k} = \frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{n}_{l_i}}\frac{\partial \mathbf{e}_k}{\partial \mathbf{p}_k^{c_j}} \quad (6)$$

The partial derivatives in camera pose estimation and the extrinsic calibration are different. It is shown in the following section.

A. The Partial Derivatives in Back Projection Function f

$$\mathbf{p}_k^{c_i} = f(\mathbf{T}, \mathbf{\Pi}) \quad (7)$$

$$\frac{\partial \mathbf{p}_k^{c_i}}{\partial \mathbf{R}} = (\mathbf{R}\mathbf{n})^\wedge(\frac{\mathbf{t}}{\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n}} - \frac{\mathbf{t}^\top\mathbf{R}\mathbf{n}\mathbf{q}_k^{c_i}}{(\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n})^2} - \frac{\bar{\mathbf{p}}^\top\mathbf{n}\mathbf{q}_k^{c_i}}{(\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n})^2})\mathbf{q}_k^{c_i\top} \quad (8)$$

$$\frac{\partial \mathbf{p}_k^{c_i}}{\partial \mathbf{t}} = \frac{\mathbf{R}\mathbf{n}\mathbf{q}_k^{c_i\top}}{\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n}} \quad (9)$$

$$\frac{\partial \mathbf{p}_k^{c_i}}{\partial \mathbf{n}} = (\frac{\mathbf{R}^\top\mathbf{t} + \bar{\mathbf{p}}}{\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n}} - \frac{(\mathbf{t}^\top\mathbf{R} + \bar{\mathbf{p}}^\top)\mathbf{n}\mathbf{R}^\top\mathbf{q}_k^{c_i}}{(\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n})^2})\mathbf{q}_k^{c_i\top} \quad (10)$$

$$\frac{\partial \mathbf{p}_k^{c_i}}{\partial \bar{\mathbf{p}}} = \frac{\mathbf{n}\mathbf{q}_k^{c_i\top}}{\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n}} \quad (11)$$

$$\frac{\partial \mathbf{q}_k^{c_i}}{\partial \mathbf{u}_k^{c_i}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\mathbf{K}^{-\top} \quad (12)$$

$$\frac{\partial \mathbf{p}_k^{c_i}}{\partial \mathbf{q}_k^{c_i}} = \frac{\mathbf{t}^\top\mathbf{R}\mathbf{n} + \bar{\mathbf{p}}^\top\mathbf{n}}{\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n}}\mathbf{I} - \frac{\mathbf{t}^\top\mathbf{R}\mathbf{n} + \bar{\mathbf{p}}^\top\mathbf{n}}{(\mathbf{q}_k^{c_i\top}\mathbf{R}\mathbf{n})^2}\mathbf{R}\mathbf{n}\mathbf{q}_k^{c_i\top} \quad (13)$$

B. The Partial Derivatives in Camera Pose Estimation

$$\mathbf{p}_k^{c_i} = f(\mathbf{T}^{c_i}, \mathbf{\Pi}^w) \tag{14}$$

$$\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{R}^{c_i}} = (-(\mathbf{p}_k^{c_i})^\wedge + \frac{\partial f(\mathbf{T}^{c_i}, \mathbf{\Pi}^w)}{\partial \mathbf{R}^{c_i}} + \mathbf{t}^{c_i\wedge})\mathbf{R}_{c_i}^{c_j\top} \tag{15}$$

$$\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{t}^{c_i}} = (\frac{\partial f(\mathbf{T}^{c_i}, \mathbf{\Pi}^w)}{\partial \mathbf{t}^{c_i}} - \mathbf{I})\mathbf{R}_{c_i}^{c_j\top} \tag{16}$$

$$\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{R}^{c_j}} = (\mathbf{R}_{c_i}^{c_j}\mathbf{p}_k^{c_i})^\wedge - (\mathbf{R}_{c_i}^{c_j}\mathbf{t}^{c_i})^\wedge \tag{17}$$

$$\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{t}^{c_j}} = \mathbf{I} \tag{18}$$

$$\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{u}_k^{c_i}} = \frac{\partial f(\mathbf{T}^{c_i}, \mathbf{\Pi}^w)}{\partial \mathbf{u}_k^{c_i}}\mathbf{R}_{c_i}^{c_j\top} \tag{19}$$

C. The Partial Derivatives in Extrinsic Calibration

$$\mathbf{p}_k^{c_i} = f(\mathbf{T}_l^c, \mathbf{\Pi}^{l_i}) \tag{20}$$

$$\mathbf{p}_k^{c_j} = \mathbf{R}_{c_i}^{c_j}\mathbf{p}_k^{c_i} + s\mathbf{t}_{c_i}^{c_j} \tag{21}$$

$$[\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{R}_l^c}, \frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{t}_l^c}] = [\frac{\partial f(\mathbf{T}_l^c, \mathbf{\Pi}_{l_i})}{\partial \mathbf{R}_l^c}\mathbf{R}_{c_i}^{c_j\top}, \frac{\partial f(\mathbf{T}_l^c, \mathbf{\Pi}^{l_i})}{\partial \mathbf{t}_l^c}\mathbf{R}_{c_i}^{c_j\top}] \tag{22}$$

$$[\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{R}^{c_i}}, \frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{t}^{c_i}}] = [-(\mathbf{p}_k^{c_i})^\wedge\mathbf{R}_{c_i}^{c_j\top} + s\mathbf{t}^{c_i\wedge}\mathbf{R}_{c_i}^{c_j\top}, -s\mathbf{R}_{c_i}^{c_j\top}] \tag{23}$$

$$[\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{R}^{c_j}}, \frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{t}^{c_j}}] = [(\mathbf{R}_{c_i}^{c_j}\mathbf{p}_k^{c_i})^\wedge - s(\mathbf{R}_{c_i}^{c_j}\mathbf{t}^{c_i})^\wedge, s] \tag{24}$$

$$\frac{\partial \mathbf{p}_k^{c_j}}{\partial s} = \mathbf{t}_{c_i}^{c_j\top} \tag{25}$$

$$\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{u}_k^{c_i}} = \frac{\partial f(\mathbf{T}^{l_i}, \mathbf{\Pi}^{l_i})}{\partial \mathbf{u}_k^{c_i}}\mathbf{R}_{c_i}^{c_j\top} \tag{26}$$

$$\frac{\partial \mathbf{p}_k^{c_j}}{\partial \mathbf{n}^{l_i}} = \frac{\partial f(\mathbf{T}^{l_i}, \mathbf{\Pi}^{l_i})}{\partial \mathbf{n}^{l_i}}\mathbf{R}_{c_i}^{c_j\top} \tag{27}$$

Let $\mathbf{p}_k^{c_j} = [X, Y, Z]^\top$, then $\epsilon_k = \frac{1}{z_k}\mathbf{K}\mathbf{p}_k^{c_j} - \mathbf{u}^{c_j}$

$$\frac{\partial \epsilon_k}{\partial \mathbf{p}_k^{c_j}} = \begin{bmatrix} \frac{f_x}{Z} & 0 & -\frac{f_x X}{Z^2} \\ 0 & \frac{f_y}{Z} & -\frac{f_y Y}{Z^2} \end{bmatrix}^\top \tag{28}$$

where

$$\mathbf{K} = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix} \tag{29}$$